5.4.3 Geodesic Dome Contours for Cylindrical Pressure Vessels © S. T. Peters 2001

The following equations and the attached programs are described in Filament Winding, Composite Structure Fabrication ,S.T. Peters, W.D. Humphrey and R.F. Foral, pp5-50-53, 2nd Ed., 1990, © SAMPE Publishers

Geodesic dome contours, generated using netting analysis procedures, have current theoretical interest. Originally, such dome contours were thought to be the optimum shape. Now, the geodesic shape is considered by some analysts to be the optimum deformed shape of the dome. Large deflection finite element analysis is used to identify the original shape which will deform to the geodesic shape under internal pressure and nozzle thrust loads. In this section, a simple iterative technique for generating the geodesic dome contour for internal pressure loading is provided.

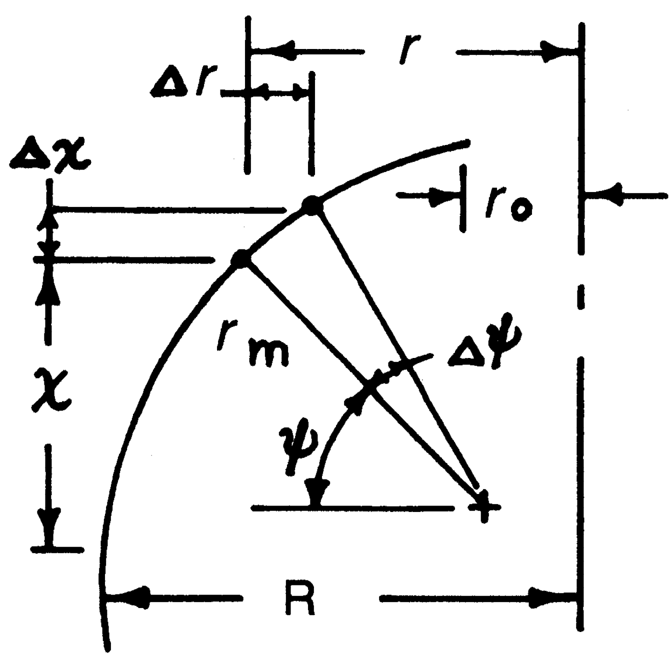

Figure 5-14.
Notation for geodesic dome contour.

Figure 5-14 provides the notation. The filament wound dome includes only [±θ fibers, except in the dome cylinder juncture region, where some hoop windings are usually extended into the dome. Thus, combining Equations [5.61] and [5.63]

$$\frac{N_h}{N_M} - \tan^2\theta \qquad \textbf{[5.68]}$$

Each fiber is assumed to carry the same tension at every point along its length, and to follow a geodesic path across the dome. Then, there is no tendency to slip, and all supportive loads pass through the axis of the vessel. If a length of the fiber is in moment equilibrium about the axis of the vessel, **r sin** θ is a constant. When θ**= 90 ^{o}, r = r_o**, the boss radius. Therefore, the local wind angle θ at radial location r is given by:

$$\theta = \sin^{-1}\frac{r_0}{r} \qquad \textbf{[5.69]}$$

If r_o is known, Equation [5.69] gives the required cylinder wind angle θ**=$\sin^{-1}$ (r/R)**, where R is the cylinder radius. If the wind angle in the cylinder is known, Equation [5.69] gives the required boss opening **r_o=R sin** θ

For a dome cap of radius r to be in equilibrium

$$N_m = \frac{p_i r}{2\cos\psi} \qquad \textbf{[5.70]}$$

And

$$\frac{N_m}{r_m} + \frac{N_h}{r}\cos\psi = p_i \qquad \textbf{[5.71]}$$

where **r_m** is the radius of curvature of the meridian. Substituting Equations [5.68] and [5.70] into Equation [5.71] and solving for **r_m** produces

$$r_m = \frac{r}{\cos\psi(2 - \tan^2\theta} \qquad \textbf{[5.72]}$$

As shown in Figure 5-14, Equation [5.72] provides the means of generating the dome contour. Starting at the dome-cylinder juncture, Ψ **= 0, r = R, x = 0,** and θ is the cylinder wind angle. Incrementing to Ψ **=** $\Delta\Psi$**.r_m** can be determined from Equation [5.72]. Then, referring to Figure 5-14, increment of r and x are given by:

$$\Delta r = r_m \bullet \Delta\psi \bullet \cos\psi \qquad \textbf{[5.73]}$$

$$\Delta x = r_m \bullet \Delta\psi \bullet \sin\psi \qquad \textbf{[5.74]}$$

For a general value of $\Psi=\Sigma\Delta\Psi$

$$r = R - \sum\Delta r \qquad \textbf{[5.75]}$$

$$X = \sum\Delta X \qquad \textbf{[5.76]}$$

At each step, the wind angle θ is determined from Equation [5.69]. The solution proceeds, step-by-step, until **$\tan^2 \theta = 2$ ($\theta = 54.7$ degree)**, an inflection point at which r_m in Equation [5.72] is undefined. Using Equation [5.69], the radial location of the inflection point is **$r = 1.22 r_o$**. In standard design procedures, a polar boss of outer radius $\geq 1.22\ r_o$ is inserted. The winding surface over the boss is commonly conical or spherical. Sometimes this region is locally reinforced and helical layers are staggered to prevent extreme thickness build up Figure 5-15 shows geodesic dome contours for r_o /R values of 0.2, 0.4 and 0.6, with corresponding cylinder wind angles (Equation [5.69]) of 11.5,23.6 and 36.9 degrees respectively. On each contour, the inflection point is marked with an "x."

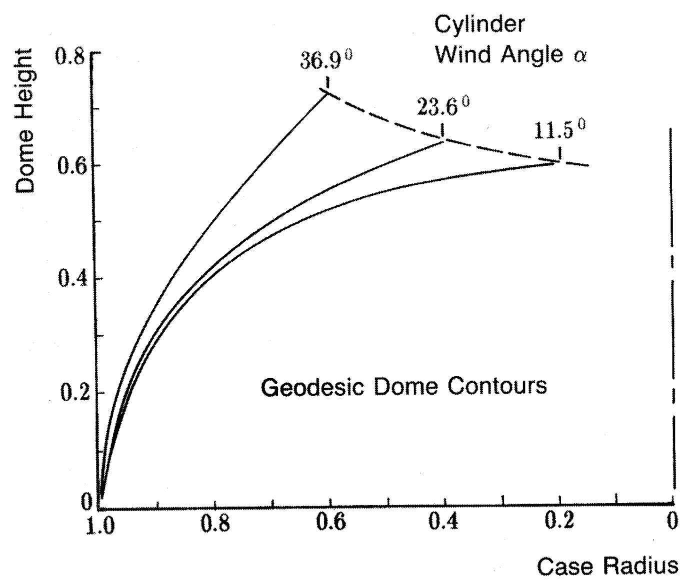

Figure 5-15.
Geodesic dome contours.